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AUTHOR(S): J. P. Shipley

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DECISION-DIRECTED MATERIALS-ACCOUNTING PROCEDURES: AN OVERVIEW*

James P. Shipley
Safeguards Systems Group
Los Alamos National Laboratory
Los Alamos, New Mexico

ABSTRACT

With materials balances taken at intervals such as six months or a year, the methods for treating materials balance data and the use of the results by safeguards decision-makers are relatively straightforward. The increasing emphasis on timely materials accounting in which balances may be drawn, say, on a daily or weekly basis, raises anew questions in these two areas: (1) What is the most effective means of extracting the maximum amount of information from the materials accounting data? (2) How should safeguards decision-makers use the results, and what, if any, impact does the decision process have on the analysis techniques?

These questions lead naturally to considering combinations of materials balances, which exposes a whole new set of concerns. For example, we must select the most appropriate combinations, which implies some consideration of possible diversion scenarios, such as abrupt or protracted. Control of the overall false-alarm rate is an important requisite of the composite procedure. Significant work has been done on loss estimators, but their role in the materials accounting decision process has only begun to be examined. Current criteria may require periodic statements with respect to materials loss; the analysis procedures must be structured to provide such information.

This paper will address these concerns, and others, by presenting an overview of the current technology. Questions still to be answered will be pointed out.

I. INTRODUCTION

The timeliness of materials accounting is governed by the frequency at which materials balances are drawn. Currently evolving requirements indicate that intervals of days to weeks will be of interest for some facilities, such as reprocessing plants. Assuming that such balances are practicable and useful, and we believe they are,[1-5] well-defined techniques for examining the materials accounting data are necessary.

For relatively infrequent materials balances, say six months or a year apart, it is reasonable and logical to treat each balance independently of the others. Statistical procedures for testing such balances for a loss of nuclear material are simple and well-known.[6,7] False-alarm and detection probabilities can be controlled (or at least bounded), and a quantifiable statement of estimated total loss (or gain) over the interval and its uncertainty can be made.

For the situation of more frequent materials balances, the single-balance methods mentioned above are no longer adequate because of the undesirable overall characteristics of the aggregate of multiple, single-balance tests. In addition, diversion scenarios covering both single and multiple balance periods must be treated. These two facts necessitate the development of new procedures to examine the materials accounting data.

It should also be clear that the statistical testing procedures must be closely coordinated with the other aspects of materials accounting, including practical constraints and safeguards requirements. It is the purpose of this paper to point out emerging requirements and survey the candidate techniques to meet those requirements.

II. THE ROLE OF STATISTICAL PROCEDURES

In the practical application of materials accounting there are three stages in which statistical procedures are important:

- detection,
- assessment,
- statement.

There are other areas where statistical methods are necessary (e.g., measurement control), but they deal primarily with obtaining the requisite materials accounting information, and we shall not treat them here.

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Detection is not the end result of materials accounting; materials accounting may detect an apparent loss of material, but the statistical procedures used for detection make no statement concerning the reason for the loss. The purpose of detection is to trigger further investigation (or to eliminate the need for it), which is intended to verify the loss and determine its cause. Defined in this way, detection can be based on purely statistical considerations, so that detection sensitivity and false-alarm and detection probabilities can generally be well-defined, at least in principle. However, at this stage, these characteristics relate to detection of a loss, not necessarily diversion. It is important that the detection stage be carried out shortly after each balance to maintain the capability for timeliness in case a loss is detected.

The detection stage for multiple-balance procedures, in contrast to that for single-balance methods, is complicated by the fact that a loss may arise from many different diversion scenarios. The detection algorithm must be capable of handling losses that occur abruptly (in one or a few balance periods) or those that accumulate over several periods. Therefore, the algorithm must collate and analyze data from past periods extending back to some specified fiducial in time. That time fiducial will generally depend on safeguards and operational requirements, but it may be updated on the basis of statistical considerations.

Because of the variety of diversion scenarios that must be accommodated, the detection stage may result in neither an investigation nor in eliminating the possibility of an investigation in the future, although such a decision may be obtained upon demand. We will return to this point later.

Assessment comprises the follow-up investigation after a loss has been detected, and it includes more extensive statistical methods for extracting the maximum amount of information from the material's accounting data. The International Working Group on Reprocessing Plant Safeguards/Subgroup 4, sponsored by the IAEA, has listed a set of steps that might be followed in such an investigation.[4] At all steps, the statistical assessment methods may guide, and be guided by, other investigative activities.

The purpose of assessment is two-fold: (1) to determine whether a loss is truly indicative of a diversion, and (2) if so, to determine what amount of material was diverted. Indications of possible accounting problems and diversion scenarios will also be of u[illegible] during the investigation. An ancillary but highly important benefit is the reduction of the ultimate false-alarm probability for the overall procedure.

Estimation of the loss is necessary during the assessment stage. All loss estimators considered to date require an assumption about the time interval over which the loss occurred. In addition, all loss estimators but the CUSUM require an assumption about the allocation of loss within the time interval. For assessment, it is straightforward to select that estimate having the greatest significance from several different estimators and assumptions on time intervals and allocations.

Loss estimation is less directly useful in the detection stage. The variety of loss scenario assumptions would dictate the need for several loss estimators to maintain an acceptable detection probability, but in such a situation it is difficult (but not impossible) to control the false-alarm probability. Therefore, it is best to minimize the need for scenario assumptions, which decreases the usefulness of loss estimators in the detection stage. However, we recognize that for some desirable testing procedures, detection and estimation are inextricably related: the CUSUM is one example, although only the time interval, and not the allocation within the interval, must be considered. This need for estimation, and its attendant assumptions, in the detection stage are at the root of many of the difficulties in designing appropriate detection testing procedures.

The final outcome of assessment is a statement of the results, for example, as specified in INFCIRC/153, para. 30.[9] In addition, a statement of non-diversion may be made immediately following the detection stage associated with the current balance if the materials accounting results indicate no loss has occurred since the last specified time fiducial. In the latter case, the assessment stage would not be necessary.

Clearly, the quality of the formal statement depends on the thoroughness of the investigation and generally cannot be specified a priori. However, the statement can, and must, be based ultimately on sound statistical methods of analyzing the available materials accounting data, including any new, quantitative information uncovered by an investigation. The quality of such a statement can be found a posteriori.

VII. PROCEDURAL REQUIREMENTS

Let us first distinguish between procedural requirements and performance requirements for testing procedures. A statistical test must be formulated to address specific questions, and it must provide answers of known quality, satisfactory or not. We may also require that answers be available on demand. Such requirements are procedural ones. On the other hand, we would like for the test to have high detection probability, low false-alarm probability, and good detection sensitivity. We would also [illegible] for the test to respond quickly. These results are performance oriented. In comparing the performance of tests, we must bear in mind the degree to which they satisfy the procedural requirements.

To ascertain the nature of procedural requirements, consider the following situation. We are given 100 material balances covering a period of two years, that is, a balance occurs

approximately once a week. Once a year, say at balances 1 and 51, a shutdown, cleanout physical inventory is taken. We have devised a statistical procedure for detection, and we ask that it meet the following criteria:

- To preserve timeliness, the opportunity to decide that a loss has occurred must be available soon after each balance.
- Losses of sufficient size occurring over any time interval, up to one-year long, for example, must be detectable. (We choose one year because that seems to be the currently accepted standard for the maximum interval.)
- A decision that diversion has or has not occurred during a time interval, and backed up by appropriate assessment procedures, may not be changed at a later time in the absence of substantial new information.
- A definitive statement of material unaccounted for and its limit of uncertainty must be available periodically, say once each calendar year at physical inventory time.
- The false-alarm and detection probabilities and the detection sensitivity must be known (or at least bounded) beforehand.

The first criterion is largely self-explanatory. However, note the implied possibility that a decision of no loss in any previous balances could be reached, eliminating those balances from further consideration.

The second criterion has several important implications. For example, the test must be sensitive to loss in any pattern, if the loss is large enough. Sometimes, this requirement is interpreted to mean that the loss detection sensitivity should be the same for all loss patterns, which might or might not be either desirable or practicable. The second criterion also implies a maximum length for the test, in this case 50 balances; it does not imply that the maximum time interval occurs between specific balances, such as 1 and 50, 51 and 100, etc., on a calendar-year basis. Otherwise, such an artificial restriction on the test procedure would allow a diverter to take one-half of a significant quantity just before balance 50, and another half just after balance 50, a time interval of perhaps two weeks, and yet have much smaller detection probability than if the diversions both occurred during the same calendar-year interval.

The third criterion protects all parties concerned from arbitrary changes in past decisions. It also allows a "fading memory" to alleviate the need in the detection stage to carry on information about which decisions have already been made.

The fourth criterion implies the ability to give a "status report" on the materials accounting results at specified times. It need not mean that a decision as to diversion or non-diversion is required at those times, although that is a possibility. If we do not require such periodic decisions and instead confine ourselves to a statement of the material unaccounted for and its uncertainty, then the continuity of the detection procedure is undisturbed, which means the fifth criterion is easier to satisfy, and the absolute requirement for a yearly assessment is eliminated. At the same time, the second criterion guarantees that the facility operator will not be held accountable for balances older than one year from the current date.

The fifth criterion states that the test must be well characterized. For a variety of reasons, insufficiently bounded or highly variable test properties are intolerable.

There are additional criteria of perhaps lesser importance but that still must be considered. For example, the test procedure must be simple enough and transparent so that it can be used in the field and the results understood by other than statistics experts. It must also be computationally feasible using the equipment likely to be at hand.

IV. SOME SPECIFIC PROCEDURES

We now consider some statistical procedures that have been shown by past experience to be useful in materials accounting. Statistical procedures for assessment will be examined first because they are more well developed and perhaps will provide insight into procedures for detection.

A. Assessment Procedures

During the assessment stages, estimation of the diversion amount and time pattern are of primary interest. Consequently, loss estimators play an important role, and several loss estimators extending over all possible time intervals will have to be used to cover all loss patterns.

Loss estimators and their properties have been discussed in many previous papers and reports,[10-21] and we shall not dwell on the details here but point out the most useful ones. The CUSUM is intuitively appealing because it estimates the total diversion in a time interval and is independent of the loss pattern within that interval. The uniform loss estimator is tailored for protracted small losses of approximately constant size, and it estimates the loss per balance over a time interval. The sequential variance estimator is designed to look for small, protracted, random losses by examining the variability in the materials balances during the time interval. In practice, these three estimators, if performed in the manner described below, are sufficient to cover almost all loss patterns. Generally, the estimation algorithms can be carried out sequentially in time, as materials balances become available, and there are advantages to doing so.[17-21]

As described in the previous example of 100 balances over a two-year period, we want to use these estimators to find the loss pattern and to

estimate the loss. We will not necessarily be interested in the biggest loss estimate, but the most significant one. Significance is normally indicated by the ratio of the loss estimate to its standard deviation, which would be equivalent to performing a statistical test if the ratio were compared to a properly defined threshold.

The method for using the loss estimators is as follows. At the current time, n, assume that balances one through n are available; balance one represents the last previous time fiducial for restarting the procedures. Each loss estimator is started at balance one and iterated through balance n, providing a loss estimate and its significance after each balance. This procedure is then repeated starting at balance two, three, etc., up to balance n. The number of loss estimate sequences for each loss estimator is n, and the number of loss estimates and significance values for each loss estimator is 1/2 n (n + 1). If the estimators are sequential in time, the entire process need not be repeated each time a new balance appears; a one-step update can be performed.

The next step is to select the most significant estimate and its corresponding sequence or pattern. It is possible to peruse the significance values individually, but for 50 balances there are 1275 of them for each loss estimator. Furthermore, the significance values, and sequences of them, are related; in fact, evolving loss patterns can be more readily discerned by examining the appreciation of the data. To do this efficiently, a graphical display technique would be helpful.

One such technique is called the alarm-sequence chart.[17-21] It is a point plot indicating those sequences of materials balances having the most significant loss estimates. Each sequence is located on a graph of initial point in the sequence vs. its final point; the plotting symbol used at that point is indicative of the significance of that sequence. Using such a device makes appraisal of the materials accounting information almost instantaneous.

Although procedures such as these are designed primarily for the assessment stage, they can also provide assistance to safeguards personnel during the detection stage. The changing materials accounting status in a facility can be followed and updated at will to provide confirmatory information supplementing the normal procedures used during the detection stage. The techniques require significant computational capability; computerized algorithms would be a great help.

B. Detection Procedures

The procedural requirements listed previously for the detection stage are stringent and severely constrain the choice of statistical tests for detection. Tests having well-defined values of false-alarm and detection probabilities are particularly desirable because the detection stage is the initiator of subsequent events, such as assessment, that may have significant consequences. However, the other criteria force consideration of procedures that are somewhat more complicated than those usually encountered in the literature. Consequently, each candidate procedure requires careful examination of its properties. Following are some possibilities.

Method 1

The techniques described under assessment procedures could be used together as a package for the detection stage, and they could be made to satisfy all five procedural criteria. A bound on the overall false-alarm probability for such a detection procedure can be obtained from a Bonferroni inequality,[22] and the methods are sufficiently flexible to answer the necessary questions at each time. However, the complete battery of techniques may be more than is necessary, and too much of a computational load, at each time step, although it may be desirable to use them every few balances just to keep close watch on the materials accounting situation.

Method 2

A simpler detection algorithm would be beneficial and perhaps, in some cases, more likely to be used in the field. Several possibilities are discussed in a related paper in these proceedings.[23] One of the best of these is the sequential test of power one. In its simplest form for uncorrelated, stationary materials balances of variance σ, the test is based on the cusum at time n and is given by[21,23]

$$\frac{c(n)}{\sigma\sqrt{n}} \geq \left\{\left(1 + \frac{1}{n}\right)\left[a^2 + \ln(n+1)\right]\right\}^{1/2}, \qquad (1)$$

where the region below the threshold (labeled NO) is the normal operating region where the sequential test continues until the threshold is crossed or the test is truncated. The realized false-alarm probability, α, is bounded by

$$\alpha \leq \frac{e^{-a^2/2}}{2\Phi(a)} \; ; \qquad (2)$$

$\Phi(a)$ is the Gaussian error function evaluated at a. Using the truncation procedures described in Ref. 23, the test may be started at an arbitrary time fiducial and truncated and restarted on demand and as appropriate. The overall false-alarm probability for such a series of tests can again be bounded by a Bonferroni inequality.

The power-one test has good performance in terms of its false-alarm probability and detection sensitivity. Its main drawback is that the truncation point is purely arbitrary; the materials accounting data do not signal the need for a restart under normal operation, which means that periodic truncation will generally be required to avoid exceeding the maximum time interval.

Method 3

This method, which is still under development, is related to Method 1 and the assessment procedure through a direct search for the "worst-case" sequence of balances. For the simplest case as in Method 2, the test is

$$C(n) - \min_{0<i<n} C(i) \underset{N_0}{\overset{N_1}{\gtrless}} T(n) \quad , \tag{3}$$

with $C(0) = 0$. The precise form of the boundary is still being investigated, but it is likely to be similar to that for the power-one test, which is based on the law of the iterated logarithm.[25,26] This test is also related to Page's test,[27-30] most closely to the version that finds the maximum change in the CUSUM from a fixed series of materials balances; previous sequential formulations have been somewhat different.

This test procedure largely alleviates the restart problem of Method 2 because the logical beginning point is the minimum of the CUSUM in the maximum time interval. The test can also be truncated and restarted as will using procedures similar to those of Method 2.

Method 4

It has been suggested[30,31] that a sensitive indicator to changes in the loss pattern, for example its onset, might be provided by examining the "innovations sequence."[32] The innovations should be zero-mean and sequentially uncorrelated for zero loss and proper algorithm formulation, and the associated test would look for departures from these assumptions.

Because the innovations sequence, by definition, contains all the available information, it should be well-suited to detecting such changes. However, it will be sensitive to any changes, including those in the loss pattern and especially those occurring in the correlated measurement errors. Therefore, follow-up investigations, perhaps like those of the assessment stage, will have to be made to separate the causes. This may require adjustment of the test threshold to achieve a suitable rate of initiation of the assessment stage. The test is still being developed.

C. Statement Procedures

As suggested previously, the statement stage should culminate in a report of the materials accounting results, that is, a statement of the materials unaccounted for and its limit of uncertainty. The statement can be made at specified points in time covering the maximum time interval, or it can be made following a detection and assessment that a significant loss has occurred. One possible method is to report the most significant results in the maximum time interval. This technique would be related to detection-stage Method 1 in that the statement of materials unaccounted for is given by

$$\max_{0<r<n} [C(r) - \min_{0<i<r} C(i)] \ , \ C(0) = 0 \tag{4}$$

for the simplest case discussed above. This statistic is not Gaussian, but it has well-defined characteristics in terms of our being able to quantify its limit of uncertainty. Note that this statistic is a global one in that it examines all the data and therefore has different properties than an isolated CUSUM.

V. CONCLUSIONS

The statistical tests used to examine materials accounting data must be specially structured to meet operating and safeguards procedural requirements. The task can be made easier by recognizing that materials accounting activities occur in three stages: detection, assessment, and statement. Statistical methods must satisfy, at least, five criteria:

- timeliness,
- flexibility with respect to time interval and loss patterns,
- stability of decisions,
- availability on demand of the materials accounting status, and
- quantifiability of performance.

Many of the techniques developed and used in the past are applicable primarily to the assessment stage, although they may be of some use during detection. This is especially true of loss estimators. Specialized algorithms designed for the detection stage are still being developed. The question of what constitutes a statement is open yet, but appropriate statistical methods are available.

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