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TITLE: EFFICIENT ANALYSIS OF DYNAMIC MATERIALS ACCOUNTING DATA

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EFFICIENT ANALYSIS OF DYNAMIC MATERIALS ACCOUNTING DATA

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ABSTRACT

Current trends in safeguarding special nuclear materials (SNM) in nuclear fuel cycle facilities portend increasing emphasis on timely collection and analysis of materials accounting data. The availability of more and better data argues for an organized framework of techniques to ensure efficient and complete extraction of information concerning possible diversion of SNM. This paper describes such a framework and presents results obtained by analyzing simulated data from a large nuclear fuel cycle facility.

I. INTRODUCTION

Materials accounting for safeguarding special nuclear material (SNM) has two important aspects: (1) the collection of materials accounting data, and (2) the analysis of materials accounting data. The collection function is a broad, highly developed subject (e.g., see Refs. 1-5 and the references therein) that we will not pursue here; in this paper we are primarily concerned with the analysis of materials accounting data.

The data-collection function is usually structured to facilitate performance of the analysis function, commonly by providing sufficient measurements of SNM so that materials balances can be drawn around selected portions of the facility on a reasonable time scale. The data, which are always corrupted by measurement errors, often appear as time sequences of materials balances, one sequence from each portion of the facility.

Therefore, the data-analysis function must operate on imperfect data that become available sequentially in time. Its primary goals are (1) detection of the event(s) that SNM has been diverted, (2) estimation of the amounts diverted, and (3) determination of the significance of the estimates. Furthermore, data analysis must search for evidence of diversion that may have occurred in any of several patterns.

II. DECISION ANALYSIS

Decision analysis[6] has been applied to nuclear materials accounting in several previous papers,[7-10] and only a brief overview is given here. Its origins lie in the U.S. space program, which has made major contributions to decision theory and systems analysis, the primary building blocks of decision analysis.

The detection and estimation functions of decision analysis are based on classical hypothesis testing and modern state-variable estimation techniques. The systems analysis portion attempts to set thresholds for the hypothesis tests in a rational fashion, for example, by using utility theory to determine desirable false-alarm and detection probabilities. In this paper, we ignore the latter problem and concentrate on detection and estimation of missing SNM.

A. Hypothesis Testing

Let $Z(N)$ represent the set of observed materials accounting data for N materials balance periods. Then, under the two mutually exclusive, exhaustive hypotheses

$$\begin{aligned} &H: \text{ no SNM is missing,} \\ &K: \text{ some SNM is missing,} \end{aligned} \qquad (1)$$

$Z(N)$ has one of the two conditional probability density functions

$$p\ Z(N)|H \quad \text{or} \quad p\ Z(N)|K ,$$

respectively. The usual statistical test consists of forming the *likelihood ratio*,[11-15] L, and comparing it to a threshold:

$$\text{If } L[Z(N)] \triangleq \frac{p[Z(N)|K]}{p[Z(N)|H]} \begin{cases} < T, \text{ accept H,} \\ > T, \text{ accept K,} \end{cases} \qquad (2)$$

where T is the test threshold. Roughly speaking, if $Z(N)$ is "enough" more likely to have occurred as a result of H being true than of K being true, then decide that H is true; otherwise, decide that K is true.

This test, for a fixed number N of points, consists of comparing the likelihood ratio to a single threshold. However, in practical situations we seldom will know when a particular diversion strategy starts or ends. Therefore, we will want to begin the test at all possible starting points and let the test itself determine when it should be terminated. This procedure also has the provident property of requiring fewer samples for a decision, on the average, than a fixed-sample-size test having similar characteristics.[13]

The sequential likelihood ratio test, also called the sequential probability ratio test or SPRT,[13] has three possible results at each decision time, rather than two:

$$\text{If } L[Z(k)] \begin{cases} \leq T_0, \text{ accept H,} \\ \geq T_1, \text{ accept K,} \\ \text{otherwise, take another observation,} \end{cases} \qquad (3)$$

and the SPRT is repeated for all possible starting points. The thresholds T_0 and T_1 can be found from the following approximation, devised by Wald, which gives useful thresholds that can be shown to be conservative. Let P_M and P_F be the desired (given) miss and false-alarm probabilities, respectively, for the SPRT. Then the thresholds are[12,13]

$$T_0 = \frac{P_M}{1 - P_F} , \qquad T_1 = \frac{1 - P_M}{P_F} \qquad (4)$$

The probability of detecting diversion, related to $1-P_M$, is called the *power* or *size* of the test; P_F is called the *significance* or *level* of the test.

B. Estimation

The original hypothesis statements, Eq. (1), are much too vague to be of practical value. More precise statements must be written in terms of the characteristics to be expected under the two hypotheses. It is this translation of physical characteristics into mathematical statements that determines the form of the estimation algorithm. It should be noted that this step *always* requires some assumption about the diversion scenario; consequently, a particular estimation algorithm and its corresponding test work best if the diversion pattern "coincides" with the hypothesis statement, K. In this regard, some algorithms are more robust in the sense that they allow a wider range of diversion scenarios while still maintaining acceptable test performance.

For most practical problems in nuclear materials accounting, linear, least-squares filtering and smoothing provide adequate estimation algorithms, particularly if, in addition, the measurement error statistics are known fairly well. The estimation can be performed on batches of data or sequentially in time. The sequential formulation has several advantages as we have already seen, and it has the further advantage that inverses of large matrices need not be calculated as they must be for batch estimation. The sequential, linear least-squares filtering algorithm using known error statistics is commonly called the Kalman filter.[16-19]

C. Test Procedure

As discussed above, we seldom will know beforehand when diversion started or how long it will last. Therefore, the decision tests must examine all possible, contiguous subsequences of the available materials accounting data. That is, if at some time we have N material balances, then there are N starting points for N possible sequences, all ending at the Nth, or current, material balance, and the sequence lengths range from N to 1. Because of the sequential application of the tests, sequences with ending points less than N have already been tested; those with ending points greater than N will be tested if the tests do not terminate before then.

Another procedure that helps in interpreting the results of tests is to do the testing at several significance levels, or false-alarm probabilities. This is so because, in practice, the test thresholds are never exactly met; thus, the true significance of the data is obscured. Several thresholds corresponding to different false-alarm probabilities give at least a rough idea of the actual probability of a false alarm.

D. Displaying the Results

Of course, one of the statistics of most interest is the estimation result. Common practice is to plot values of the test statistic, with symmetric error bars of length twice the square root of its variance, vs the materials balance number. The initial materials balance and the total number of materials balances are chosen arbitrarily, perhaps to correspond to the shift or campaign structure of the process. For example, if balances are drawn hourly, and a day consists of three shifts, then the initial materials balance might be chosen as the first of the day, and the total number of materials balances might be 24, covering three shifts. This choice is for display purposes only; the actual testing procedure selects all possible initial points and sequence lengths, and any statistic may be displayed as seems appropriate.

The other important results are the outcomes of the tests, performed at the several significance levels. A tool called the alarm-sequence chart,[7-10] has been developed to display these results in compact and readable form. To generate the alarm-sequence chart, each sequence causing an alarm is assigned (1) a descriptor that classifies the alarm according to its false-alarm probability, and (2) a pair of integers (r_1,r_2) that are, respectively, the indexes of the initial and final materials balance numbers in the sequence. The alarm-sequence chart is a point plot of r_1 vs r_2 for each sequence that caused an alarm, with the significance range of each point indicated by the plotting symbol. One possible correspondence of plotting symbol to significance is given in Table I.

TABLE I

ALARM CLASSIFICATION FOR THE ALARM-SEQUENCE CHART

Classification (Plotting Symbol)	False-Alarm Probability
A	10^{-2} to 5×10^{-3}
B	5×10^{-3} to 10^{-3}
C	10^{-3} to 5×10^{-4}
D	5×10^{-4} to 10^{-4}
E	10^{-4} to 10^{-5}
F	$<10^{-5}$
T	~ 0.5

The symbol T denotes sequences of such low significance that it would be fruitless to examine extensions of them; the letter T indicates their termination points. It is always true that $r_1 \leq r_2$ so that all symbols lie to the right of the line $r_1 = r_2$ through the origin. Examples of these charts are shown in the section on results.

III. SOME USEFUL TESTS

We desire a test, or tests, that is most effective against the extremes of diversion patterns, from a large, single theft to a series of small, uniform thefts. It is unlikely that one test would be equally effective against all diversion scenarios. Following are three tests and estimation algorithms that cover the possibilities quite adequately.

A. Uniform Diversion Test (UDT)

The statement of the alternative hypothesis for the UDT is that the diversion during each materials balance period is constant in time at some unknown level. Minimum-variance, unbiased estimates of the unknown level and the inventory at each time are given by the Kalman filter described in Ref. 9, which also gives a method for treating correlated measurement errors correctly. Similar, but less general formulations are reported in Refs. 20-23.

The UDT suffers from the restrictive assumption on the diversion scenario. The test works very well against long-term, multiple theft, but offers little improvement over testing individual materials balances for single theft.

B. Sequential Variance Test (SVT)

One characteristic to be expected when diversion is present is a larger materials balance error variance than when there has been no diversion. The SVT uses two Kalman filters, each similar to that for the UDT, to calculate the materials balance error variances under the hypothesis and alternative. The result is roughly equivalent to a sequential formulation of the well-known F test for variances. The corresponding assumption on the diversion scenario is that the diversion during each materials balance period is a Gaussian random variable having constant mean and variance, which are *a priori* unknown. Maximum-likelihood estimates of the mean and variance are computed sequentially from the likelihood ratio as the data are received.

As with the UDT, the SVT provides estimates of both the missing material and the inventory at each time. However, the total amount of missing material over the test interval is also computed by subtracting the last inventory estimate from the first inventory measurement and adding in the intervening net transfers. This estimate of the total diversion is more indicative of the materials accounting situation. Note that the alarm-sequence chart refers not to the missing-material estimates, but to a possible shift in materials balance error variance.

The diversion pattern assumed for the SVT is much less restrictive than that for the UDT because almost any set of diversions could have been drawn from a white, Gaussian, random process, even if the diversion were constant or intermittent. The only real restrictions are that the mean and variance be constant over the test interval. However, the test procedure covers all possible intervals, so that this assumption is less restrictive than it might seem.

A similar estimation algorithm was described in Refs. 20-23, but no procedures for obtaining the diversion mean and variance were given. In addition, it was not clear what decision test was to be used.

C. Smoothed Materials Balance Test (SMBT)

Stewart[24] noted earlier that better (smaller variance) materials balances could be drawn if past data were used to calculate the beginning inventory of the current materials balance. He proposed the equivalent of a Kalman filter, assuming no diversion before the current time, for performing the calculation. This technique can be extended if one is willing to consider deferred decisions. That is, if we have data from N materials balance periods and we wish to compute the materials balance at time k, where k lies between 1 and N, then we can (1) run a "forward" Kalman filter from time 1 to k to estimate the kth beginning inventory, (2) run a "backward" Kalman filter from time N to k + 1 to estimate the kth ending inventory, and (3) subtract the result of (2) from that of (1) and add the intervening transfer measurement to find the smoothed materials balance at time k based on the data from time 1 to N. The procedure can also be done for any number of intervening materials balance periods, and it includes Stewart's method as a special case.

Significant improvements in materials balance uncertainties may be obtained with the SMBT; the price is a delayed decision. Care must also be taken when applying the test to intervals in which several diversions may have occurred; that situation violates the assumption (no diversion) on which the filters are based and can cause incorrect materials balance estimates.

IV. SOME REPRESENTATIVE RESULTS

The following results are taken from Monte Carlo simulation studies of a large chemical separations facility.[2] Figures 1 and 2 show the outcomes of the UDT, SVT, and SMBT for no diversion over a two-day period of one-hour balances. For reference, Fig. 1 also shows the standard materials balance (Shewhart) chart. On all charts of statistics, the horizontal marks are the values of the statistics, and the vertical lines are $\pm$ 1σ

error bars. The paucity of alarms is clear in all cases; in fact, no alarm-sequence chart is given for the SMBT because no alarms were generated.

Figure 3 shows the results of the UDT and the SVT for a uniform-diversion of 75 g/balance (1.8 kg/day). This level is $< 0.1\sigma$ for a single materials balance, yet both the UDT and SVT readily detect the diversion after about 24 balances (1 day). The SVT is also slightly more indicative of missing material than the UDT (compare Figs. 3b and 3d). The UDT estimate is 73 g/balance, and the SVT estimate is 3.58 kg total for two days. The results of the SMBT are not shown for this case.

Figure 4 gives the outcomes of the SVT and SMBT for the case of two diversions of 1 kg each occurring at balances 19 and 34. Again, the indications are definite, although the SMBT shows more clearly that there were two diversions. However, the SVT gives the better estimate of the total missing material, 2.2 kg total at time 48. For clarity, only the single materials balance alarms are given for the SMBT.

V. CONCLUSIONS

There are many other kinds of tests available that make use of other characteristics of materials accounting data. Suboptimal least-squares methods and nonparametric tests that do not depend on detailed knowledge of the measurement error statistics immediately come to mind. These additional techniques are currently being developed, but we feel that the three tests discussed here would probably remain as the most useful and generally effective. Such a battery of tests and display formats, unified by the framework of decision analysis, provides a highly comprehensive and efficient package for analyzing materials accounting data, especially from near-real-time accounting systems.

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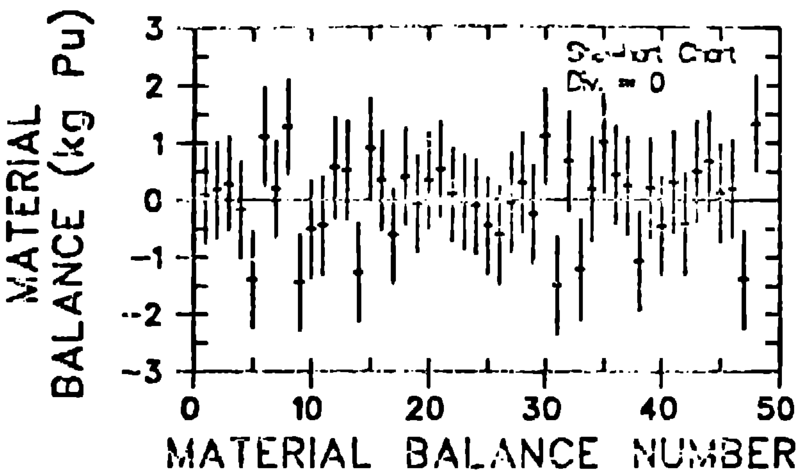
FIGURE CAPTIONS

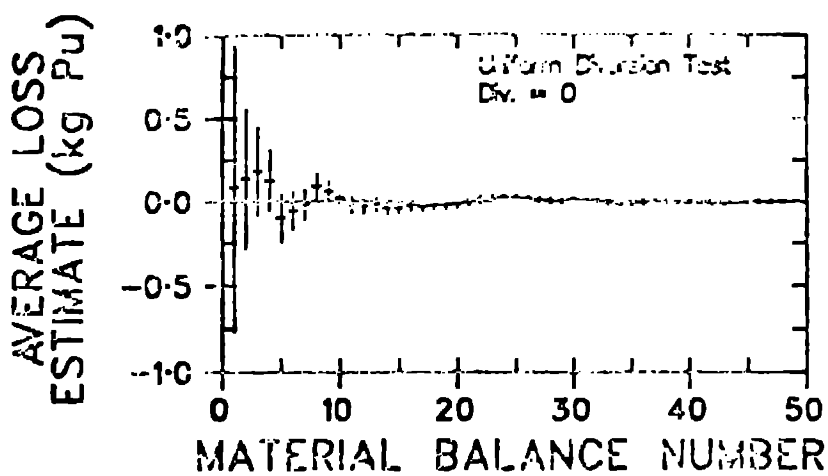
Fig. 1. Test results for no diversion: (a) Shewhart chart, (b) UDT loss estimate, (c) UDT alarm-sequence chart.

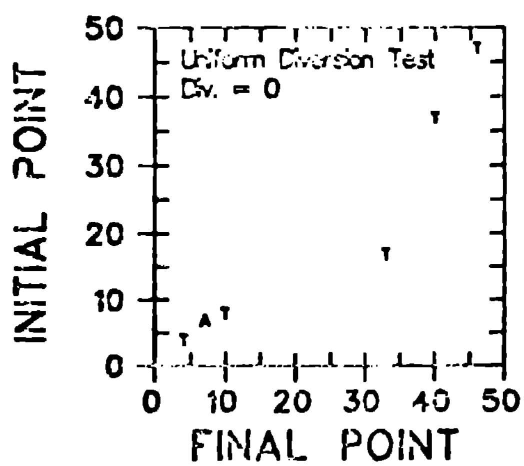
Fig. 2. Test results for no diversion: (a) SVT total loss estimate, (b) SVT alarm-sequence chart, (c) SMBT materials balance chart.

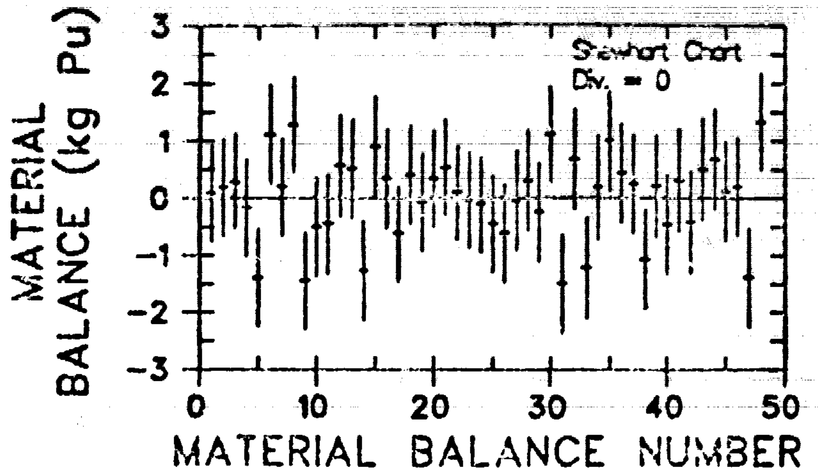
Fig. 3. Test results for 75 g/balance diversion: (a) UDT average loss estimate, (b) UDT alarm-sequence chart, (c) SVT total loss estimate, (d) SVT alarm-sequence chart.

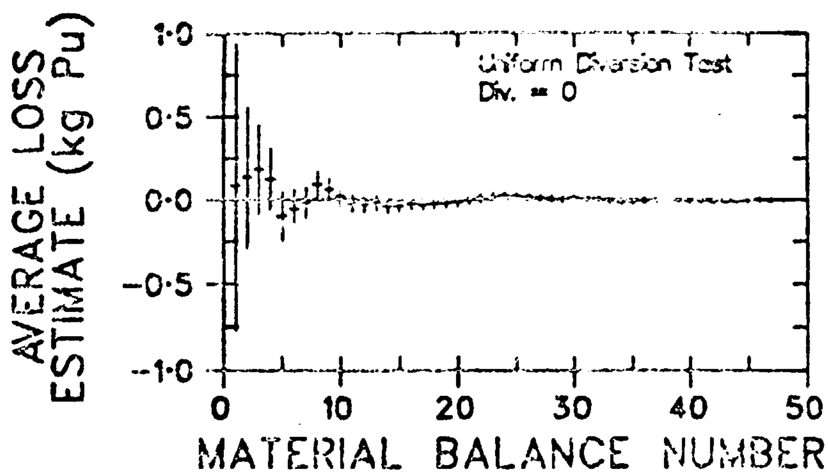
Fig. 4. Test results for 1 kg diversion at t = 19,34: (a) SVT total loss estimate, (b) SVT alarm-sequence chart, (c) SMBT materials balance chart, (d) SMBT alarm-sequence chart.


MATERIAL BALANCE (kg Pu)
Div. = 0
MATERIAL BALANCE NUMBER


AVERAGE LOSS ESTIMATE (kg Pu)
Div. = 0
MATERIAL BALANCE NUMBER


INITIAL POINT
Uniform Diversion Test
Div. = 0
FINAL POINT


Shewhart Chart
Div. = 0
MATERIAL BALANCE (kg Pu)
MATERIAL BALANCE NUMBER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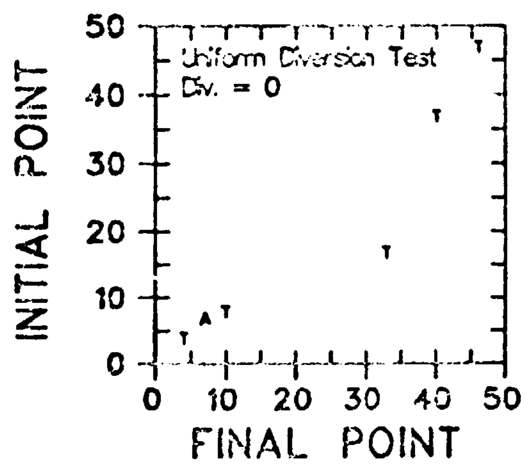
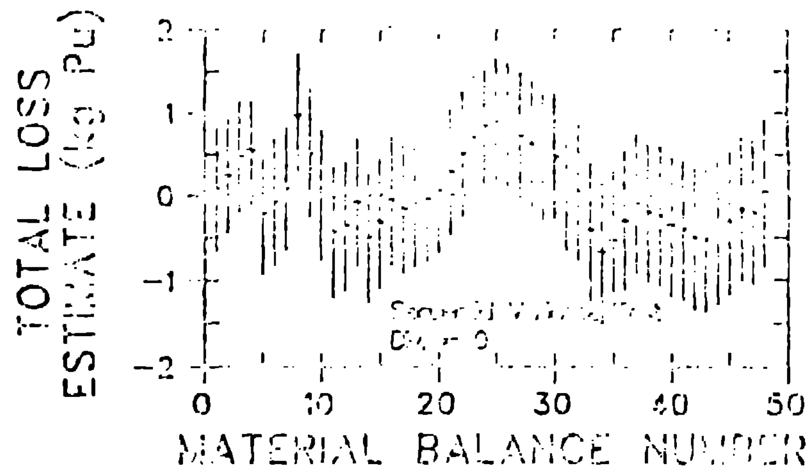
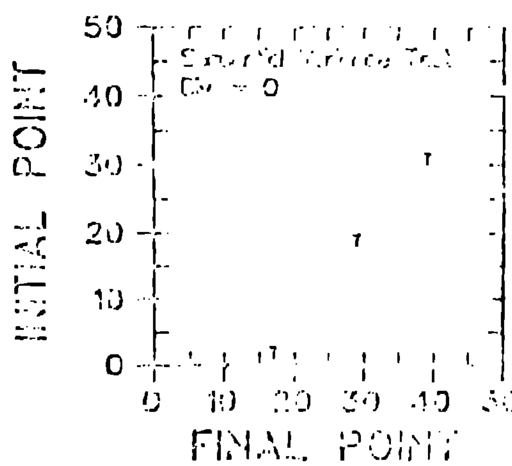
AVERAGE LOSS ESTIMATE (kg Pu)
MATERIAL BALANCE NUMBER

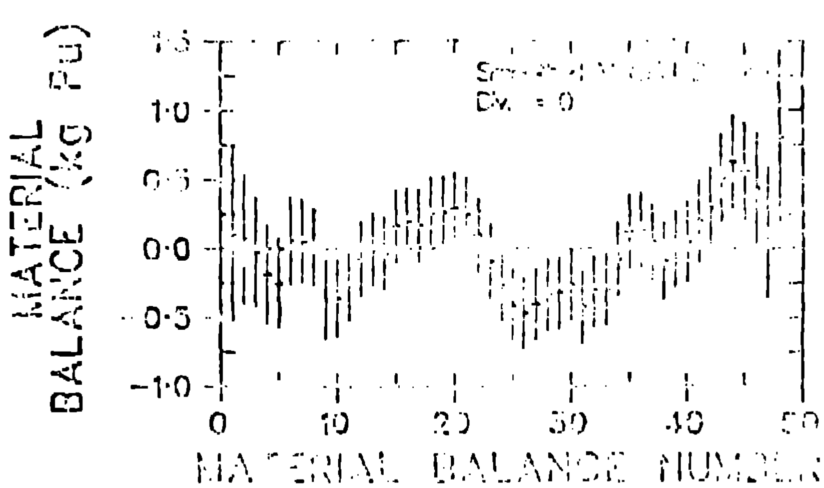

Uniform Diversion Test
Div. = 0
INITIAL POINT
FINAL POINT



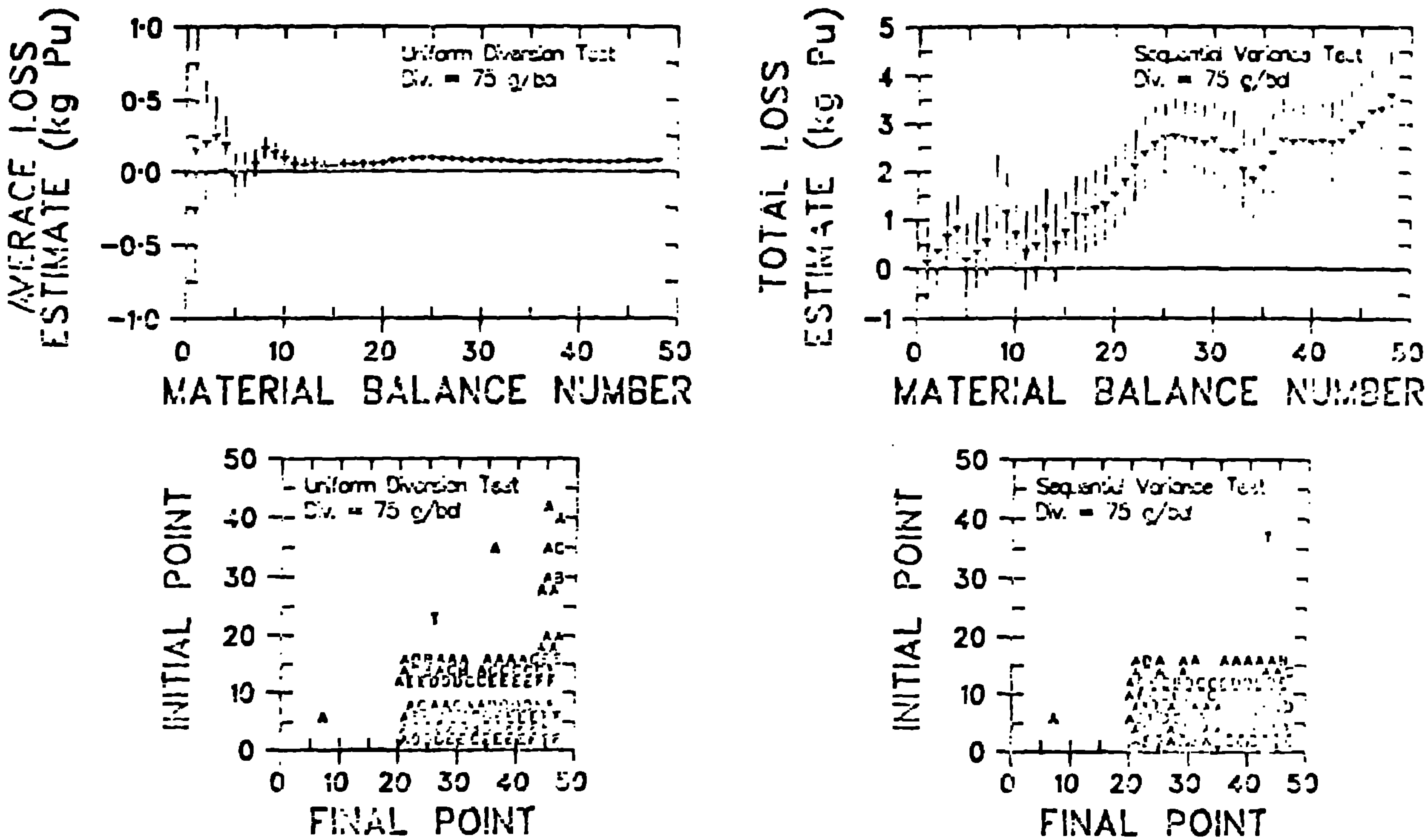
TOTAL LOSS ESTIMATE (kg Pu)
2
1
0
−1
−2
0
10
20
30
40
50
MATERIAL BALANCE NUMBER



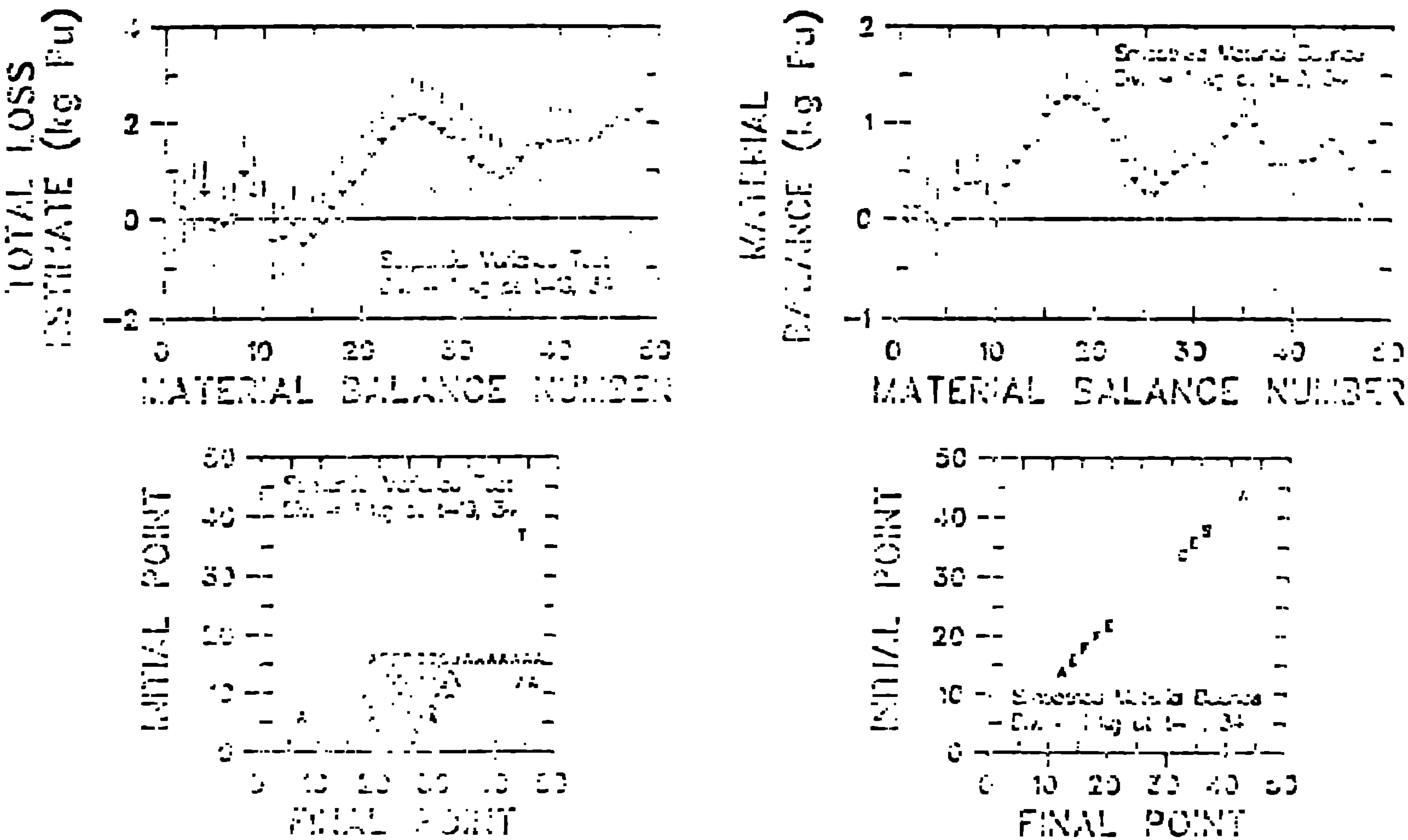
INITIAL POINT
50
40
30
20
10
0
0
10
20
30
40
50
FINAL POINT



MATERIAL BALANCE (kg Pu)
1·0
0·5
0·0
-0·5
−1·0
0
10
20
30
40
50



Uniform Diversion Test
Div. = 75 g/bd
AVERAGE LOSS ESTIMATE (kg Pu)
MATERIAL BALANCE NUMBER
Sequential Variance Test
Div. = 75 g/bd
TOTAL LOSS ESTIMATE (kg Pu)
MATERIAL BALANCE NUMBER
Uniform Diversion Test
Div. = 75 g/bd
INITIAL POINT
FINAL POINT
Sequential Variance Test
Div. = 75 g/bd
INITIAL POINT
FINAL POINT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MATERIAL BALANCE NUMBER
MATERIAL BALANCE (kg Pu)
MATERIAL BALANCE NUMBER
INITIAL POINT
FINAL POINT
INITIAL POINT
FINAL POINT