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TITLE HADRON SPECTRUM FROM THE LATTICE

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HADRON SPECTRUM FROM THE LATTICE

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ABSTRACT

Considerable progress has been made in the last year in deriving the spectrum from QCD in the quenched approximation. I review these results and show that we are close to getting results with 10% errors for the proton to rho mass ratio. I give a status report on QCD calculations with dynamical fermions being done by various groups. While these calculations are still exploratory, we have reached the stage where realistic simulations can be contemplated.

INTRODUCTION

A realistic calculation of the hadron spectrum will be the first demonstration of our ability to get reliable results from Lattice QCD. This calculation is necessary in order to show that QCD is the correct theory of strong interactions. Only then will the more important and predictive calculations of matrix elements be justified. To achieve this goal, we have to proceed incrementally. We need to first get definitive results in the quenched approximation for realistic values of the quark masses and lattice momenta. Only then can we systematically investigate the effect of quark loops. With the present state of algorithms, I believe that with a tera-flop, large memory machine we will be in a position to do definitive quenched calculations and at the same time quantify the effects of dynamical quarks. So, in this review I will summarize the progress made to date and highlight the directions one needs to explore to reach the goal.

Quenched simulations provide an important reference point. Most of the software and techniques for numerical measurement carry over unchanged to the real theory. Since full QCD calculations are still a factor of ≈ 1000 slower, it is expedient to clean up the techniques and understand systematic errors in quenched simulations. Pure gauge theory is confining and asymptotically free, so it contains the qualitative essence of the real world. Further, chiral symmetry can be studied in the quenched approximation: the chiral behavior of observables can be derived and checked. It is therefore very important to get statistically significant numbers for the $n_f = 0$ world so that one can there after systematically examine the effect of quark loops in $n_f = 2,3,4$ simulations. It may very well turn out that the quenched approximation is good for certain observables; unfortunately this justification can only come a-posteriori. On the other hand for certain features like the glueball spectrum we definitely need to understand mixing with meson states before we can approach an experimentalist with a hard number.

The notation used below is as follows: the gauge coupling is defined by $\beta \equiv 6/g^2$; the quark mass m_q is given by κ for Wilson fermions, and the physical mass of the strange quark is denoted by m_s. I use a superscript v (d) for valence (dynamical) quarks when ever necessary. The number of dynamical quark flavors is given by n_f. The spatial volume is denoted by N_s^3 and the temporal size by N_t such that the lattice size is $N_s^3 \times N_t$. The effective mass of a particle is defined to be $M(t) = \log\left(\frac{\Gamma(t)}{\Gamma(t+1)}\right)$, where $\Gamma(t)$ is the 2-point correlation function. The desired answer is the asymptotic value as $t \to \infty$.

In keeping with the long term approach of lattice calculations to (a) get very accurate quenched results, (b) systematically investigate the effect of quark loops as a function of the quark mass and the number of flavors, and (c) do realistic calculations at weak coupling and at small quark mass on large lattices, I first summarize the status of quenched calculations for mesons and baryons. Then I present the status of calculations with dynamical fermions. Note that these calculations are still preliminary because the masses of dynamical quarks used in the update are still fairly heavy. This review covers new results, most of which has been presented for the first time in preliminary versions at this conference. For calculations done prior to 1988, I refer you to the comprehensive reviews by M. Fukugita [1] A. Ukawa [2] and E. Marinari [3] .

1. Quenched Spectrum.

Calculations of the spectrum in the quenched approximation began about 8 years ago. The touchstone for measuring progress has been the ratio R of the proton mass to the rho mass. This has in the past (until 1988) came out consistently high, usually > 1.6. The measurements were, however, carried out at heavy quark mass ($m_q \geq m_s$), on smallish lattices and the statistics were often inadequate. The situation has changed considerably in the last year due to improved measurement techniques and significantly more computer time.

In the real world we know two data points; (a) $R = 1.5$ for infinitely heavy quarks and (b) $R = 1.22$ for physical quarks. In between, where all lattice results lie, we can partly bridge the gap using phenomenological models. For heavy quarks, we can use potential models while for light quarks one should use the chiral Lagrangian. Fitting these models to experimental data we can deduce the expected behavior as a function of quark mass. This is shown in fig. 1a and 1b as dark lines. The two lines have very different curvature and neither model is expected to do well in the region $m_\pi/m_\rho \sim 0.5$. I analyze the collective data from large lattice simulations in the quenched approximation against this background. These results are taken from three groups –– APE [4] , Iwasaki *etal.* [5] , and the staggered Collaboration [6] . The data is shown in fig. 1a for Wilson fermions and fig. 1b for staggered fermions using the APE invariant mass plot.

The mass plots show a very significant trend; the ratio R decreases with increasing β. Already, at $\beta = 6.0$ the data fall on or even slightly below the phenomenological curves. If this trend continues as β is increased, then the quenched theory number will fall below the experimental value. This possibility should raise the eyebrows of the advocates of a large strange quark contribution to the mass of the proton. It has been conjectured that up to $400 MeV$ of the proton mass comes from the strange sea. This is based on the mismatch between the experimental value of the pion-nucleon σ term ($50 - 60 MeV$) and first order SU(3) breaking analysis ($\approx 26 MeV$) [7] [8] . Clearly, what we measure in the quenched approximation are the masses with no sea quark contribution to any state. So the quenched ratio can lie on either side of the real world, depending on how large the sea quark contribution is to the proton versus the rho!

A second important feature of the data is the lattice size dependence. This can be seen in fig. 1a for Iwasaki *et.al.*'s data at $\beta = 5.7$ and for APE data at $\beta = 6.0$. Ironically, R decreases as the lattice size is increased for APE data but increases in Iwasaki's! This lack of understanding and control over finite size effects is perhaps the biggest reason why the quenched spectrum data has been so murky until recently. Let me provide a rule of thumb approach to how large the lattice should be to extract a meaningful behavior of M versus β. For $\beta = 6.0$ and $m_q \approx m_s$ we require at least $20^3 \times 40$ lattices. For different β and m_q, this size should be scaled as follows: increase each dimension as $1/a$ when increasing β and as $1/m_\pi$ when decreasing m_q.

A measure of how well the lattice can reproduce hyperfine interactions is the splitting between the Δ and proton as a function of the quark mass. The experimental number for the ratio of the mass of the Δ to the proton is 1.31. The Wilson fermion data show that this ratio is ≈ 1 for heavy quarks and increases as the quark mass is decreased The ratio increases to ≈ 1.2 at the smallest quark mass in the APE data at both $\beta = 5.7$ and 6.0. While this trend is encouraging, our enthusiasm has to be tempered by the fact that we do not know what the quenched result should be. As data for still smaller values of the quark mass becomes available, we should make fits to a $1/(m_1 m_2)$ mass dependence, with some appropriate ansatz for the constituent quark mass m_i. The signal for the Δ with staggered fermions is still too poor to extract any numbers, so the comparitive consistency check cannot be made.

The detailed comparison between Wilson and staggered results is made in the section 3. Here let me just state the conclusion: the large lattices data show that results using Wilson fermions and staggered fermions start to come together only for $\beta \geq 6.2$. At $\beta = 5.7$ the deviations are substantial.

2. Desideratum.

quark sources:

It is not surprising that much can be gained by improving the interpolating field operators used for probing physics. E. Marinari provided compelling evidence for multi-origin quark propagators and smeared operators in last year's review [3]. Let me at the outset stress that

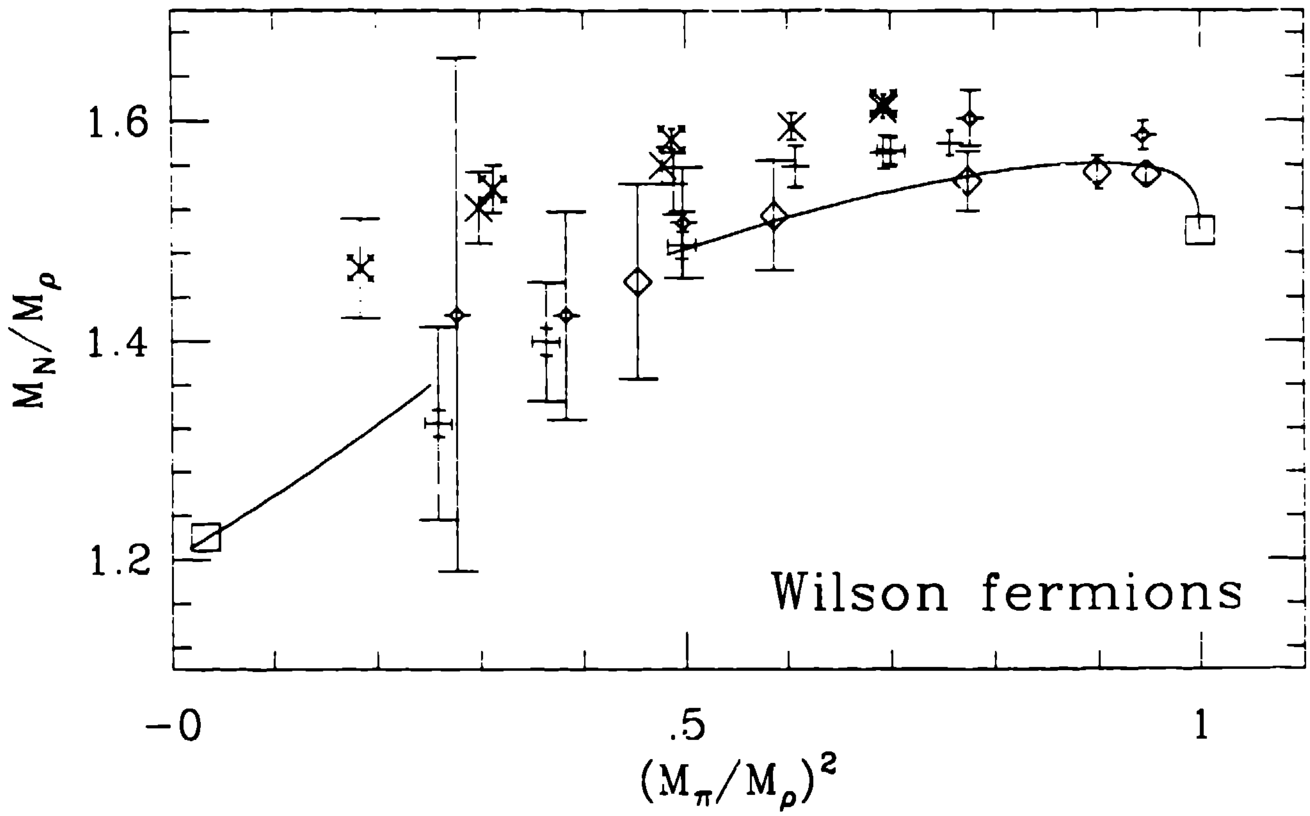

Fig. 1a: *The APE mass plot for Wilson fermion data. Data at $\beta = 5.7$ is from the APE collaboration on $12^3 \times 24$ lattices ($\times$) and $24^3 \times 32$ lattices (fancy $\times$) [4]. The data at $\beta = 5.85$ is from Iwasaki et.al. on $16^3 \times 48$ lattices ($\diamond$) and $24^3 \times 60$ lattices (fancy $\diamond$) [5]. The data at $\beta = 6.0$ is also from the APE collaboration on $18^3 \times 32$ lattices (+) and $24^3 \times 32$ lattices (fancy +)[4].*

there is no one source that will give close to optimum results for all physics. We should do calculations with different physically motivated sources on a wide variety of observables so that a clear picture of improvement is obtained by making careful comparisons. The key here is details and precise tests. Let me focus on the physics motivation for "multi-origin" sources and "smeared" operators.

Iwasaki *et.al.*'s work using point quark sources shows very clearly that, especially for baryons, the asymptotic mass can only be extracted at very large separation t. Unfortunately, in the chiral limit the signal becomes increasingly poor at large t. Thus we need to build in the wave-function into the interpolating field operators. Since, the physical size of a hadron, is fixed by the confinement scale (determined by the value of β), we expect the wavefunction to be fairly insensitive to the quark mass when $m_q \leq \Lambda_{QCD}$. Thus it may not be necessary to do a lot of tuning to improve the wavefunction. The gain in the signal with even the simplest wall source, however, has been remarkable.

Two kinds of quark sources have been explored for Wilson fermions. The APE group has used *cube* sources *i.e.* a unit source at each point in a spatial cube of certain size. They find that the signal improves even further on using multiple cube sources on a time slice. In this case each cube acts as a source with a smeared wavefunction, and the separation between the

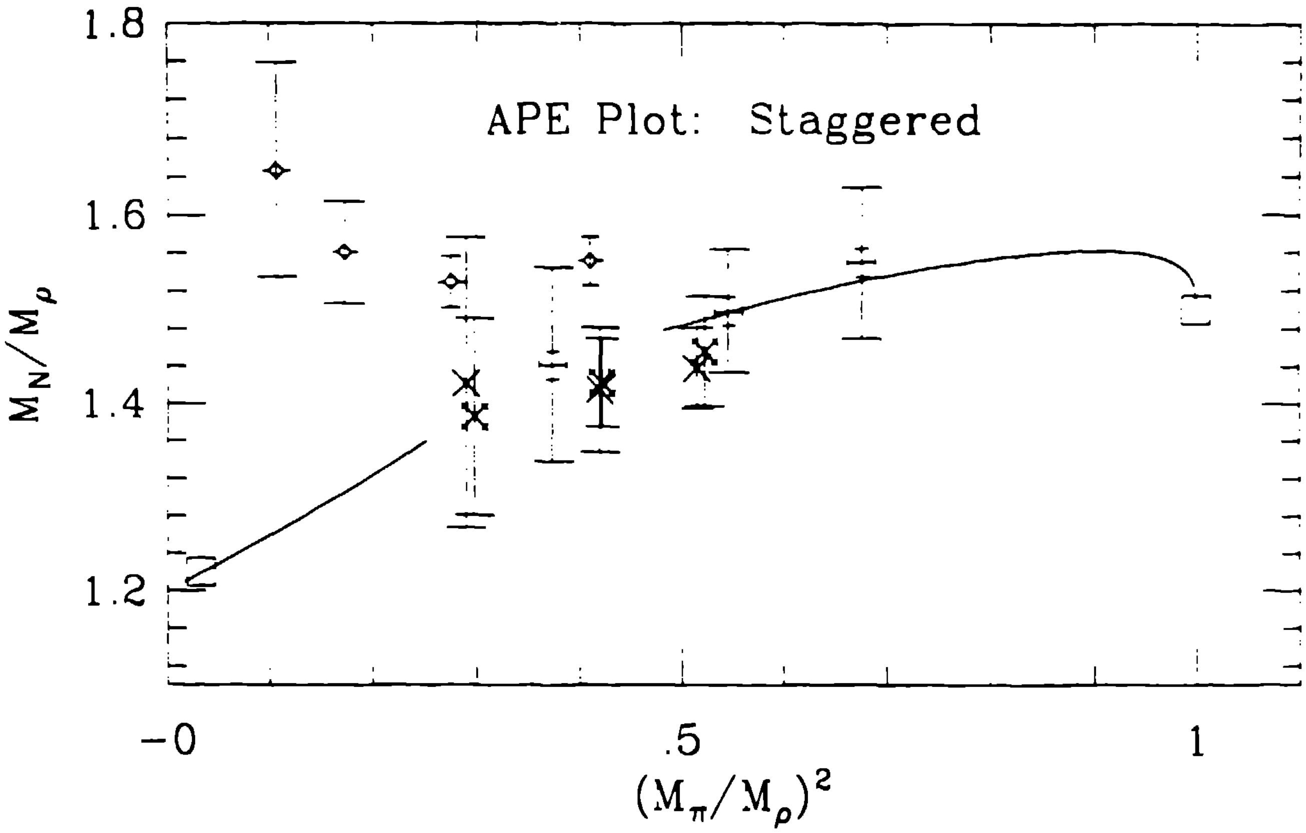

Fig. 1b: *The APE mass plot for Staggered fermion data. Data at $\beta = 5.7$ is from the APE collaboration on $24^3 \times 32$ lattices ($\diamond$) [4]. The rest of the data is from the Staggered Collaboration: $\beta = 6.0$ on $16^3 \times 40$ lattices ($\times$) and $24^3 \times 40$ lattices (fancy $\times$), and $\beta = 6.2$ on $18^3 \times 42$ lattices ($+$) [6].*

cubes reduces noise between the multiple sources. A source in which the cube is the whole time slice, is called a *wall* source. In such constructions of cube sources, the gauge links connecting the multiple source points are ignored, so it is essential to gauge fix the source time slice to Coulomb gauge. If one further uses extended operators at the sink without including links in a covariant fashion, then the whole lattice should be fixed to Coulomb or Landau gauge depending on whether or not the operators are extended in the time direction.

The Wuppertal group uses the solution of a three dimensions scalar field equation with a delta function origin on a time-slice as the source for the quark propagator. The scalar mass is tuned to give a reasonable size for the wavefunction. This solution provides an exponentially damped wave-function and is my favorite for the following reason: it is gauge covariant and corresponds to a wavefunction of the lowest radial state. In the non-relativistic limit, the states we measure have zero orbital angular momentum and are not radially excited. If we assume that each of the quarks inside the hadron moves freely, then, ignoring hyperfine interaction, the solution to the scalar field equation is the wave function we want. In the Wuppertal construction the local quark propagator S_F is replaced by

$$S_F(x,x') \rightarrow SS_F(x,x') = O(x,y)S_F(y,z)O(z,x')$$

where the wave-function of a quark inside a periodic box, O, is included both at the source and at the sink. Note that SS_F has the same gauge covariance property as the local propagator. Thus, the hadron correlators are constructed in exactly the same way.

For staggered fermions, the two basic sources that Greg Kilcup has developed are 1) a wall source and 2) a source with all even points on the time slice set equal to +1 and all odd points set to -1 [6]. The meson correlators are then constructed by first making the 3 distinct bilinear combinations from these two quark propagators calculated with different sources. Further, consider the four linear combinations of these bilinears. The basic point here is that these four meson correlators obtained as a result of the above combinations have different projections on to various spin-parity channels. One has to make a library of all possible spin-parity channels, and empirically determine which correlator gives the best signal for a given state. The plethora of channels is further enlarged by considering local and non-local operators at the sink point. Thus one can estimate the mass for all states (16 pions *etc.*), and test whether flavor symmetry is restored. For baryons, all three quark propagators that are contracted together are generated with the same source. The two sets of correlators corresponding to different sources are then added together (having taken care of overall signs) to improve statistics. The LANL collaboration finds that the best signal for baryons comes from such correlators and by using non-local operators at the sink point. Obtaining the same mass from many different channels improves the confidence level of the estimate.

Improved non-local operators:

Multi-origin sources are intended to improve the overlap with the wave-function at the source. Further improvement can be made by using an interpolating operator that matches the wavefunction at the sink time-slice also. Both improvements independently help saturate the exponential fall-off of the 2-point correlation function by a single particle state. The Wuppertal construction is symmetrical between the source and the sink, though in principle one could use a different mass in the scalar field equation at the two ends. The staggered signal for hadron correlators is usually much poorer than Wilson fermions. In addition, there is contamination from the opposite parity particle whose correlator has a (1^{-t}) prefactor. The work of LANL collaboration shows that by using non-local operators, the projection on the oscillating channel can be vastly reduced.

Finite Size Effects:

Systematic studies show that finite size effects are large. Unfortunately, the data do not map on to asymptotic scaling formulas yet. The good news is that the signal with larger N_s compensates to a large extent the increase in the CPU time necessary to simulate the larger volume *i.e.* the errors are roughly constant for runs of constant CPU time. The use of extended sources and operators allows us to extract masses at smaller time separation. This is especially relevant since as $m_q \to 0$ the signal becomes poor at large separations. Thus, we need lattices with large N_s in which case $N_t \approx 2N_s$ suffices. In addition, I refer you to the review by S. Sharpe for a discussion of chiral logarithms and finite volume effects on them.

Error reduction via boundary conditions:

Different boundary conditions are used in an attempt to reduce finite N_s or finite N_t effects. For example, the use of either Neumann or Dirichlet b.c. in time allow us to extract the effective mass from larger separations. Second, by combining quark propagators or hadron correlators, calculated using periodic and anti-periodic boundary conditions, we can reduce finite N_s errors. The drawback of these tricks is that while they work in practice, they do not provide a way of extrapolating to infinite volume results. Combining propagators with periodic and anti-periodic boundary conditions in time directions is identical to solving the Dirac equation on a periodically doubled lattice. On the doubled lattice, the correlators have useful signal up to separation N_t. If one further enlarges the lattice, then the quality of the signal is further improved. Consider solving the Dirac equation on a quadrupled lattice with p.b.c.. The correlators have a cosh form centered about $2N_t$, and the two arms are not statistically independent. For the region $N_t < t < 2N_t$, the time slice N_t acts as the source of the state with the wavefunction generated dynamically. Thus, as long as N_t is long enough to damp out all higher states, the signal in the second quadrant will be dominated by a single exponential. This trick should certainly be used with dynamical fermion lattices, where update time is much larger than that for measurements.

Making Fits:

One of the big problems with lattice calculations is evaluating the reliability of a result. In the case of hadron spectrum calculations we are looking for small differences, for example R changes from 1.5 ($m_q = \infty$) to 1.22 ($m_q = 0$). Thus, it is vital to remove as much subjective analysis (or extrapolations) as possible; for example, in extracting masses from fits to hadron correlators. The procedure I recommend is to first determine the location of the plateau in the effective mass plot and to then make a single exponential fit to the data in this region. As a merit of goodness, one should specify the number of time slices that constitute the plateau for each state. The second case is the extrapolation in m_q. In the new data that I have presented here linear fits do not work very well. Therefore, less emphasis should be put on the extrapolated value, and more on the actual numbers displayed on either the APE or Edinburgh mass plot along with a systematic error analysis that takes into account correlations.

Finite lattice spacing a:

Present calculations at $\beta = 6.0$ have $a \approx 0.1$ fermi. To show that we have control over systematic errors due to lattice discretization, mass ratios should remain constant as a function of β for a scale change of at least 2. Tests of scaling show that this translates into showing constant mass-ratios over the interval $6.0 < \beta < 6.5$ for the pure gauge theory [9] . The equivalent interval for the $n_f = 2$ theory is likely to be $5.7 < \beta < 6.1$ for light quark masses *i.e.* $m_q^d < m_s$.

Improved actions:

The lattice actions can be modified by adding any number of irrelevant operators *i.e.* operators of dimension ≥ 5 which vanish as $a \to 0$. The goal is to improve scaling and/or get rid of bad scaling behavior of operators which can arise due to a lattice artifact. Unfortunately, so far we have not achieved much success in getting improved scaling by adding terms to either the gauge or fermion action. There are some tantalizing hints of improvement from the Wuppertal group [10] , but no new results for the spectrum have been presented in the last year. A new direction, motivated by matrix elements calculations, is discussed by G. C. Rossi [11] . Lot more work needs to be done, however, to systematically follow through an improvement program. This possibility should be explored in the coming years.

3. Comparison of Wilson and staggered data.

We expect that the effects of chiral symmetry breaking (Wilson) or flavor symmetry violation (staggered) to decrease as $a \rightarrow 0$. Unfortunately, a quantitative evaluation of the dynamic restoration of these symmetry's requires detailed calculations. Our present guess is that these symmetries are restored to $\sim 10\%$ for $a < 0.1$ fermi. Because of the large differences between Wilson and Staggered fermion formulations, a check on lattice calculations is to demand consistency between the two results. Let me make this explicit by showing a bad, an intermediate and a good situation *i.e.* by comparing the large lattice data at $\beta = 5.7$, 6.0 and 6.2. The Wilson data at 6.2 is from the old LANL calculation [12] , while the staggered data is from the new analysis in which our estimate of the baryon mass is significantly reduced [6]. The values of the constant and the coefficient of the term linear in m_q for the fits $M = C + Sm_q$ (M^2 for the pion) are given in table 1. The Wilson quark mass is defined as $m_w = \log\left(1 + 0.5(\frac{1}{\kappa} - \frac{1}{\kappa_c})\right)$.

At $\beta = 5.7$, both the constant and the linear term is ~ 2 times larger for staggered fermions than for Wilson. Thus mass ratios are consistent between the two formulations, but individual masses are not. At $\beta = 6.0$, the linear term is ~ 3 times larger but the factor in the constant term is only ~ 1.2. What is amazing is that by $\beta = 6.2$, the constants are equal, so in the chiral limit the two formulations give the same lattice results.

The factor of ~ 2 in the coefficient of m_q may very well be a problem with the definition of mass in the Wilson formulation. Since the theory has no chiral symmetry, there is no unique definition of mass. We could, by fiat, demand consistency in the slope of m_π^2 between the two formulations to define the Wilson mass. It is only in the continuum limit that this definition has to agree with m_w as defined above. The present data show no measureable change in going from $\beta = 5.7$ to $\beta = 6.2$. We may therefore not be able to do better than a factor of two in estimating the physical quark masses.

4. Simulations with Dynamical fermions

It would be fair to summarize that so far all calculations with dynamical fermions should be regarded as exploratory. The three reasons why present results cannot be considered quantitative are: (a) the gauge coupling used is not in the scaling region, (b) the lattice volumes used are small, and (c) the quark mass is $m_q^d \geq m_s$. The degree to which these factors are present in a particular calculations, I leave to you to judge. This review is a classification of the

$\beta = 5.7$				
	C_W	C_S	S_W	S_S
M_π^2			2.8	6.8
M_ρ	0.53	0.9	1.8	3.4
M_N	0.79	1.4	3.3	5.2
$\beta = 6.0$				
	C_W	C_S	S_W	S_S
M_π^2			1.9	5.8
M_ρ	0.33	0.40	2.0	5.6
M_N	0.43	0.50	4.4	11.3
$\beta = 6.2$				
	C_W	C_S	S_W	S_S
M_π^2			1.4	4.3
M_ρ	0.27	0.30	2.4	4.2
M_N	0.44	0.40	3.9	8.6

Table 1: The mass parameters $M = C + Sm_q$ for Wilson (W) and staggered (S) fermions at $\beta = 5.7$, ,6.0, and 6.2. For the pion, the fit is to M^2. The uncertainty in these estimates is $\sim 20\%$, coming from the type of fit made.

calculations the various groups are doing rather than a presentation of hard numbers, with particular emphasis on the physics issues.

The only progress made in the update algorithms in the last year has been some understanding of critical slowing down in HMCA, and in matrix inversion algorithms [13] . These issues have been reviewed by P. Mackenzie at this conference. The HMCA [14] is the only algorithm which has a built in internal check on the accuracy required of all matrix inversions in the update. The simple yet sensitive test is to make sure that the change in the action along the trajectory does not depend on the inversion accuracy. HMCA has two drawbacks: 1) it can only be used to simulate mutiples of two Wilson flavors or four staggered flavors and neither of these represent the real world. 2) The update is slow and the configurations show auto-correlations times of many hundred trajectories. A typical example is shown in fig. 2 for 1×1 loops from a run on 12^4 lattices at $\beta = 5.4$ and $\kappa = 0.161$ done at Pittsburgh Supercomputing Center by the LANL collaboration. It remains to be seen whether long distance observables have similar correlations. In my opinion, the limitation on the number of flavors one can simulate using HMCA is sufficient reason why the present class of approximate algorithms should be explored further. For, in the end we may be forced into a two step approach: first we match say $n_f = 2$ hybrid results with those from HMCA in order to make sure that the step-size errors are smaller than some prescribed error criterion, and then simulate the real world of two light and one strange quark with the hybrid algorithm.

Results for 2 flavors of Wilson fermions:

The LANL collaboration has undertaken a systematic study to quantify the effects of quark

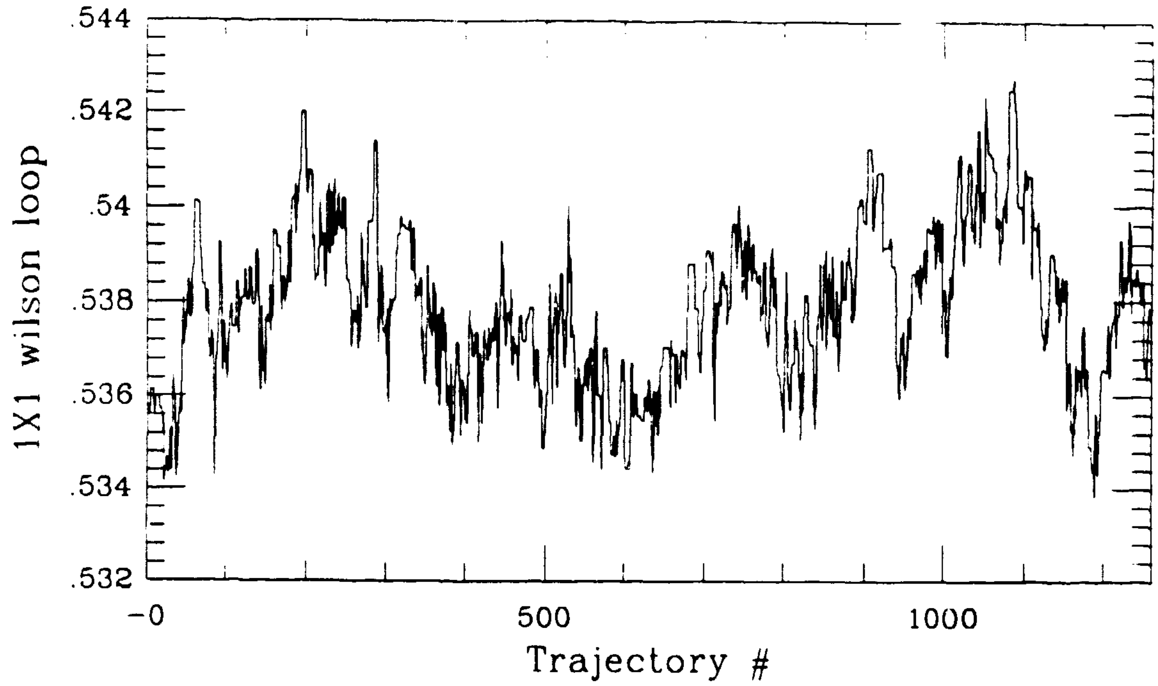

Fig. 2: *Time history of* 1×1 *Wilson loop at* $\beta = 5.4$ *on* 12^4 *lattices for the* $n_f = 2$ *Wilson fermion action.*

loops for $\beta = 5.3, 5.4, 5.5, 5.6$ using a variety of quark masses. A goal of this study is to calculate hadron masses at as low a quark mass as possible with a given lattice size so that we can look for trends as β is increased. This program is similar to the quenched case except for an additional complication. We do not know a-priori the value of the dynamical quark mass at which effects of quark loops will show up above statistical and systematic errors. From the Wilson loop data (screening in the $q\bar{q}$ potential) and preliminary spectrum analysis, a first estimate of the value of κ at which we observe the effects of quark loops is given in Table 2. It is not surprising that the numbers correspond to $m_q^d \leq m_s$. With present computer power we are barely able to simulate at $m_q \approx m_s$ on 16^4 lattices, so we will most likely not be able to quantify the effects of dynamical fermions until the advent of tera-flop machines.

β	κ_{eff}	κ_c
5.3	≥ 0.167	0.1685
5.4	≥ 0.162	0.1635
5.5	≥ 0.160	0.161
5.6	≥ 0.157	0.158

Table 2: The critical parameters for $n_f = 2$ *Wilson fermions.* κ_{eff} *is a rough estimate of* κ *at which the effects of dynamical fermions* **start** *to show up. Errors are suppressed since these estimates are preliminary.*

In fig. 3 I show the world data for $\beta = 5.5$. The older calculation (✲) is by Fukugita *et.al.* who used a $9^3 \times 36$ lattice and a second order Langevin update algorithm [15] . The rest of the data are from the LANL group on 16^4 lattices generated using the Hybrid Monte

Carlo Algorithm. This calculation is being done on the Connection Machine 2. If we linearly extrapolate the HMCA data taken at $\kappa = 0.158$ and 0.159 to $\kappa = 0.160$, we see a disagreement with the Langevin data. We will soon have data at $\kappa = 0.16$ with HMCA, corresponding to $m_q \approx m_s$, which will allow us to make the deviation quantitative. It will, however, not be possible to resolve whether the deviation is due to the approximate nature of the Langevin algorithm or due to finite volume effects.

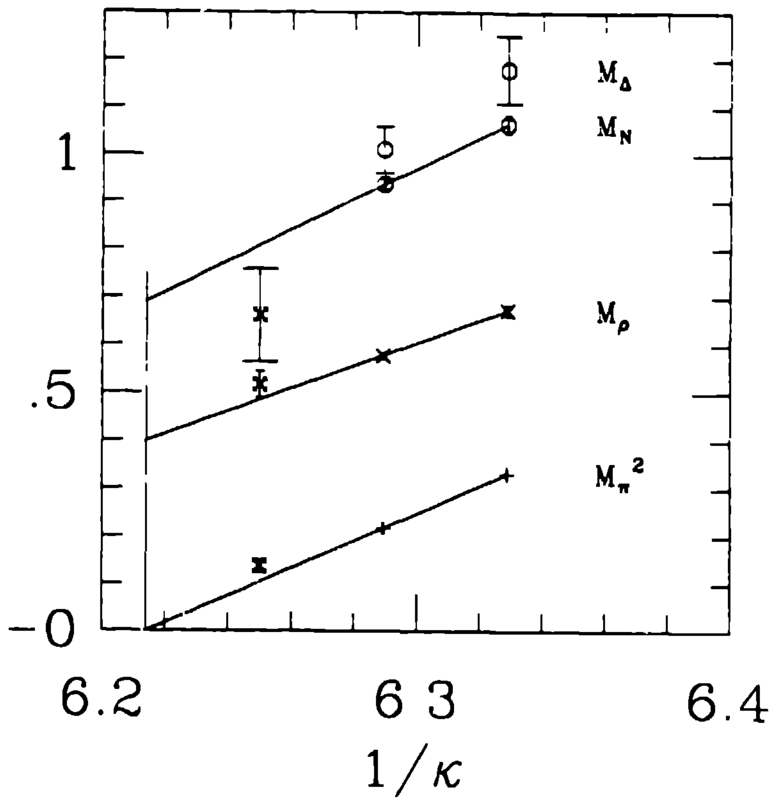

Fig. 3: *Masses in lattice units versus $1/\kappa$ for $n_f = 2$ Wilson fermion simulations at $\beta = 5.5$.*

The ratio R extracted from the data in fig. 3 is very similar to the quenched case for heavy quarks. In this regard we have not made much progress. The mere fact that we can generate configurations that incorporate the effect of dynamical fermions using an exact algorithm on lattices as large as 16^4 and $m_q^d \approx m_s$ represents a significant step forward. Let me conclude with an estimate of computer time needed to simulate a world with $n_f = 2$ Wilson fermions and $m_q^d = m_s$ on a $16^3 \times 32$ lattice. To generate 20 decorrelated lattices will require 1 **Gigaflop** year. This is clearly within our reach already.

In an earlier calculation done at stronger coupling (β =5.3) [16] we did find evidence for a large effect of sea quarks on masses. This feature is not seen when Wilson propagators are calculated on background configurations generated with $n_f = 2$ dynamical staggered flavors as discussed below.

Due to the fact that fermion update is slow, most present calculations use lattices that are doubled or tripled or quadrupled in the time direction for calculating quark propagators. For example, the LANL collaboration doubles the 16^4 lattices to $16^3 \times 32$, while HEMCGC collaboration doubles or quadruples their 12^4 configurations. As discussed previously, such periodic replication allows one to extract a much more reliable value for the mass.

High Energy Monte Carlo Grand Challenge:

The HEMCGC collaboration has generated 12^4 lattices at $\beta = 5.6$ with $n_f = 2$ flavors of staggered fermions. They use the hybrid algorithm for update and have data for the two values $m_q^d = 0.025$ and 0.01. Preliminary results of their high statistics study for the spectrum have been presented at this conference by K. Bitar (Wilson valence quarks) and W. Liu (staggered) [17] . Their study presents a comparison of the convergence of the effective mass between point sources and cube sources (unit source on half a time slice). The result is an overwhelming reaffirmation in favor of the cube source.

The staggered results are very encouraging. The ratio R is 1.45(3) for $m_q^v = m_q^d = 0.025$ and 1.31(4) for $m_q^v = m_q^d = 0.01$. These numbers need to be confirmed on larger volumes.

Their effective mass data for Wilson valence quarks show a very worrisome trend: there is essentially no dependence on the dynamical quark mass. The same is not true of the staggered results. Also, they find that given the staggered values for the nucleon, ρ and π, there is no single value of κ for which the Wilson results are the same. The deviation is enormous as shown in fig. 4. They conclude, based on this difference, that $\beta = 5.6$ is not in the scaling region. I would like to suggest, in addition, a more dangerous possibility that the dynamical staggered quarks do not couple with proper strength to the external Wilson quark propagators. The symmetries of the intermediate states are of staggered fermions. Staggered symmetries are very different from those of Wilson fermions at $\beta \sim 6.0$. These same symmetry considerations make interpretation of staggered species as n_f flavors with some masses inconsistent as discussed in section 5. Therefore, it is highly unlikely that the coupling between external Wilson states and intermediate staggered states can be viewed as Wilson with Wilson along with a simple redefinition of m_q^d. This would explain why there is essentially no m_q^d dependence and also why different Wilson states, because of their different spin contractions, do not correspond to some equivalent staggered quark mass.

Mass-T_c Collaboration:

Present results from this inter-continental group are taken from Born *et.al.* [18] . They update $16^3 \times 24$ lattices using HMCA with $n_f = 4$ staggered flavors. Their preliminary conclusion is that all physical behavior is qualitatively correct and similar to results obtained in the quenched approximation. The only place their present calculation shows a measurable effect due to quark loops is in the screening of the $q\overline{q}$ potential. They propose to extend their calculations to smaller quark masses to check for loop effects on the spectrum.

Pseudo-fermion Update:

The calculations by Potvin *et.al.* [19] show that PF always under-estimate the effect of fermions. The relevant parameter controlling the systematic error due to the approximate nature of the algorithm is $\rho/(m_\pi a)^2$, where ρ is the size of the hit matrix *i.e.* $U_{hit} = \exp(1\rho\vec{\lambda}\cdot\vec{\theta})$. Because

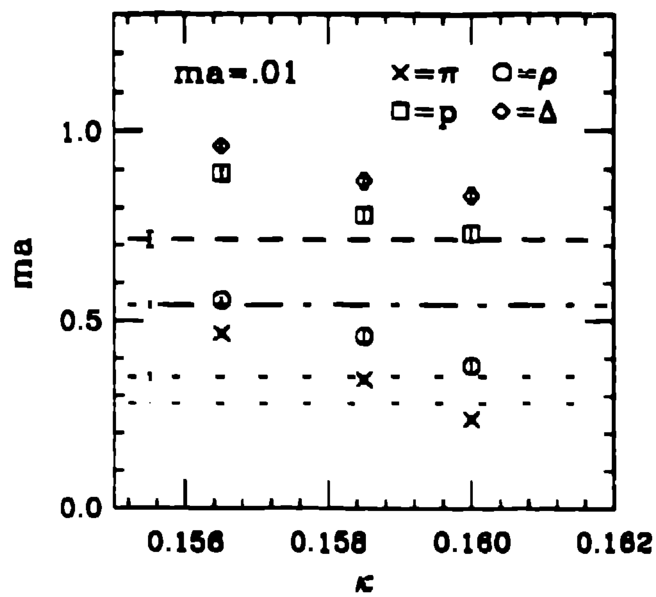

Fig. 4: *Comparison of data for Wilson versus staggered valence quarks by the HEMCGC collaboration[17]. The dashed lines are the staggered results for π, $\tilde{\pi}$, ρ and nucleon mass.*

of this, the step size has to be very small and consequently the the auto-correlations are many thousands of sweeps through the lattice. There have been many variations of the PF algorithm proposed and some still being developed [20] . My feeling is that if you fix all the problems then the speed of the algorithm is at best similar to the hybrid update[18]. This point was also emphasized by Weingarten in LATTICE88 where he showed that all these algorithms are essentially equivalent.

5. Technical issues that still haunt us.

Staggered flavors:

In all quenched simulations we find evidence of staggered flavor symmetry breaking. The weakest coupling at which we have reliable data for the "other" π's and ρ's is $\beta = 6.0$, and there the effect is $\approx 10\%$. Away from the continuum limit, it is not clear how to interpret this flavor doubling. We cannot in any simple manner regard them as four favors with possibly distinct effective masses. One way to see a problem with such an interpretation is as follows: the 16 lattice pion states break up into 8 representations (4 one dimensional and 4 three dimensional) under the hypercubic group, which implies up to 8 distinct masses, and not the 10 distinct combinations that can be constructed from 4 flavors. The number of distinct bilinear combinations reduce to 6 if two of the quarks have degenerate mass. In principle this is a possibility if there are two degenerate representations, however, no combination can be formed that matches the degeneracy of the states. The counting can be made to work if 3 staggered flavors have degenerate masses, in which case there are only three distinct masses. Note that whatever the scenario, it has to be true at all a. This requirement kills the possibility that 3 of the flavors are degenerate because in the present data, at say $\beta \approx 6.0$, we see at least four distinct pions. So it is not clear how to interpret staggered flavors at current values of β. This has an important consequence for update with dynamical fermions; one cannot rigorously specify the number of flavors or their mass by which to label a given simulation. It is only if we assume that the theory has n_f degenerate flavors can we specify m_q^d in physical units by using the lattice scale evaluated from the spectrum data extrapolated to $m_q^v \rightarrow 0$ (same procedure as in the quenched approximation).

The mass of the quarks cannot be specified in physical units for Wilson fermions either. So, this lack of definition of mass is a problem for both types of fermions. For the spectrum we are only interested in the chiral limit defined by $m_\pi = 0$ (double extrapolation with $m_q^d = m_q^v \rightarrow 0$) where only mass ratios matter, and the definition of quark mass is a non-issue. A potential problem with staggered fermions arises if we cannot specify the number of quark flavors and their mass ratios simultaneously. Then unlike the quenched approximation, where one can effectively project on to the Goldstone pion, this lack of clear statement on the number of flavors and their masses becomes an important unresolved problem for full QCD simulations.

$\rho \rightarrow \pi\pi$

The threshold for ρ decay is $m_q \approx m_s/4$. Such a calculation has to be done on a huge lattice which has the correct small but non-zero momentum. Lattice calculations, by present estimates, are at least 10 years away from such simulations. So it will be hard to show whether there is a major change in the ratio R at threshold. By then we will have much more experience from matrix element calculations with the functional form necessary to include $\rho \rightarrow \pi\pi$ decay in the mass fits.

$\mathbf{f}_\pi$

Our ability to calculate pseudoscalar decay constants, $f_\pi \ldots f_B$ accurately is important for two reasons. (1) We can use f_π to set the lattice scale if and when it becomes hard to use the ρ due to the decay $\rho \rightarrow \pi\pi$. (2) Knowing f_B gives us a handle on $B\overline{B}$ mixing. Another important issue concerning these decay constants is the presence of chiral logarithms (meson loop contributions) as explained by Steve Sharpe. These logs give large, $\sim 20-30\%$, corrections in f_π and f_K which are not present in the quenched approximation. So up to $\sim 50\%$ of the experimental value $f_K/f_\pi - 1 = 0.22$ cannot be determined in the quenched approximation. Since, this is one place where we roughly know the size of the change expected on using dynamical lattices, it should be used to caliberate realistic simulations.

Finite step size errors:

Hybrid and Langevin update algorithms have finite step size errors. A control over the ensuing systematic errors from configurations generated with approximate algorithms is in practice only a matter of computer time. By making detailed comparison between say hybrid and HMCA algorithms for $n_f = 2$ Wilson update as a function of the step size, one can develop an empirical understanding of the errors. I hope to see considerable more data in the coming years and possibly a theory to fit it.

6. Conclusions.

I hope I have convinced you that we are at the treshhold of real progress in quenched simulations. Over the next few years, we will systematically be able to reduce statistical and systematic errors. Update with quark loops is slow and present calculations are basically exploring algorithms. Real progress will come only when we can simulate light quarks.

The spectrum calculations do not have any theoretical loose ends. Over the next years progress will come from better algorithms, improved measurement techniques and a lot more computer power. So let me conclude with a peek at the future. We need a dedicated *tera-flop* machine with large memory to simulate 128^4 quenched lattices for the range of coupling $\beta = 6.2 - 6.5$. We will then be able to measure the hadron spectrum and matrix elements for $m_q \approx m_s/12$, and check scaling over this range for $m_q^v \approx m_s/5$. Also, the smallest lattice momentum for $1/a = 3GeV$ is $\frac{2\pi}{La} \approx 150\, MeV$. With these parameters we should have unequivocal quenched results to within 5% accuracy. With this same machine we will be able to simulate 32^4 lattices using HMCA for both Wilson and staggered fermions. This lattice size is large enough to quantify the effect of quark loops at $m_q^d \approx m_s/3$. From there on all improvements will be real progress towards getting hard numbers. I, therefore, anticipate that 1993-95 will be very exciting years.

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